# RAISING THE THERMAL RESISTANCE OF MULTIPLE-CONTACT PLATE STACKS

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Multiple-contact stacks of plates either sprayed with manganese dioxide or perforated have better thermal characteristics than stacks of solid "pure" plates. Under vibrating loads, however, the insulating characteristics of the former and the mechanical strength of the latter are degraded. Furthermore, the manufacturing cost of either type of stack is quite high [1].

The mechanical strength of laminated metal stacks can be retained and their thermal resistance can be increased at the same time, according to studies performed with the aid of a test apparatus shown in [2], if the surfaces of the laminations are made rougher by knurling or by shot blasting. The smaller number of contact points between contiguous surfaces and the thus greater number of areas of high thermal resistance results in a higher thermal contact resistance, while the increase of the dislocation density in the crystal lattices of the material through cold working – in the case of knurled or shot blasted plates – results in greater strength of the surface layers [3]. Both factors reduce the overall area of thermal contact. Finally, the higher thermal resistance of such stacks can also be attributed to the lower thermal conductivity of the material as a result of more intense electron and phonon scattering at pit defects produced by cold working [4].

The thermal resistance of Kh18N10T-M stainless-steel plate stacks, 0.10, 0.50 mm thick, after surface knurling and shot blasting, is calculated by the method which has been proposed in [5]. In the case of an elliptical shape with two contact spots and with an elongation ratio $\varkappa = b_e/h_e < 1$ ($b_e$ and $h_e$ denote the minor and the major semiaxes of the ellipse, respectively), under assumptions similar to those in [6], the following expression is obtained for calculating the thermal resistance:

$$R_{el} = \frac{1}{\pi i \bar{\lambda} k_0 \operatorname{tg}^2 \varphi \operatorname{sh}^2 \eta_0 \sqrt{1 + \operatorname{sh}^2 \eta_0}} \sum_{k=1}^{\infty} b_{2k-1} P_{2k-1}(i \operatorname{sh} \eta) [1 + P_{2k-1}(\cos \varphi)].$$

A comparison between calculated and experimental values of the thermal resistance for such stacks indicates close agreement.

## NOTATION

$\bar{\lambda} = 2\lambda_1\lambda_2/(\lambda_1 + \lambda_2)$ is the reduced thermal conductivity of the material of the plates, taking into account the boundary temperatures $T_1$ and $T_2$;

$k_0 = \sqrt{b_e^2 - h_e^2}$;

$\operatorname{sh}\eta_0 = h_e/\sqrt{b_e^2 - h_e^2}$;

$\varphi$ is a parameter deriving from the boundary condition for the Laplace equation in curvilinear coordinates;

$b_{2k-1} = (4k-1)/P'_{2k-1}(i \operatorname{sh}\eta_0) \int_1^{\cos\varphi} P_{2k-1}(t)\sqrt{t^2 + \operatorname{sh}^2 \eta_0}\, dt$;

$P_{2k-1}(\xi)$ are Legendre polynomials.

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# INDUCTION PERIOD DURING IGNITION OF A PARTICULATE PHASE IN A HOT GAS

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A particulate phase uniformly dispersed in a gas occupies a certain volume. All particles are of the same size. At the surfaces of the particles there occurs a heterogeneous reaction between the excess oxidizer also contained in the gas and the substance of the particles. It is assumed that the particles are sufficiently small that the entire process is limited by the chemical kinetics of the reaction only. If the actual volume of the particles and the mean distance between them are much smaller than the gas volume and the characteristic macroscopic dimension, respectively, then the gas–particles mixture can be simulated by a continuous medium [1].

The induction period for such a particulate phase is determined by the approximate method.

The basic premise in this method is that the rate of the chemical reaction depends very much on the temperature and, consequently, the entire process of heating the particles up to ignition can be broken down into two characteristic stages:

a) inertial heating of the particles from the initial temperature $T_0$ to the temperature $T_d$ corresponding to the point at which the temperature–time T(t) curve bends;

b) chemical heating of the particles at a temperature $T(t) > T_d$; this stage is either quasisteady (if $T_d < T_i$ [2]; $T_i$ denotes the flash point of a single particle) or adiabatic (if $T_d > T_i$).

In the analytical solution, the two stages of heating are made to join at the bending point of the curve, within an accuracy determined by the continuity of the first derivatives, as in [3].

The use of the method of successive approximations to produce the same results as the method of joining solutions is considered. The solution which describes inertial heating of the particles by the gas is taken as the zeroth approximation.

It follows from this solution that for $T_d < T_i$ the principal contribution to the dimensionless induction period is the time of quasisteady chemical heating of the particles from $T_d$ to $T_i$, whilst for $T_d > T_i$ it is the time of inertial heating from $T_0$ to $T_d$ (the time scale is based on the period of adiabatic induction of particles at temperature $T_d$). If $T_d = T_i$, then the dimensionless induction period is minimum. A comparison with numerical solutions obtained on a computer indicates a satisfactory agreement with the adiabatic and quasisteady approximation.

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# CRITICAL CONDITIONS IN THE STEADY-STATE OF THE IGNITION OF A PARTICULATE PHASE

V. I. Lisitsyn and V. N. Vilyunov

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A particulate phase uniformly distributed in a gas is simulated by a continuous medium with volume sources of heat (reacting particles). When the cluster of particles in the gas occupies a narrow gap of finite width at an incandescent wall, then the critical conditions for its ignition are defined as those under which no steady-state mode can be sustained in the gap [1].

The critical length $L_{cr}$ as a function of the particle radius (at a constant mass concentration) has a maximum, which is explained by the effect of two opposing trends: comminution increases the total surface area of particles and lowers their temperature (reduces the heat generated per unit surface area).

The maximum $L_{cr}(r)$ corresponds approximately to $r_m = 0.89\,\bar{r}$, where $\bar{r}$ denotes the radius at which the ignition temperature of a single particle $T_i$ [2] is equal to the temperature $T_w$ of the incandescent wall. (The ignition temperature of the cluster, as a function of the particle radius, has a maximum as a result of the interaction of these factors [2].)

Combustion will occur at any gap dimension if $r > \bar{r}$, which, in physical terms, means that now the particles near the incandescent wall ignite independently of the thermal processes taking place in the gas.

Raising the temperature $T_w$ results in combustion. If the gap dimension is smaller than the critical gap dimension corresponding to the ignition temperature of a single particle of a given radius ($L < L_{cr}(T_i)$; moreover, combustion will occur as the temperature $T_w$ rises because conditions become unsteady in the particles rather than in the system as a whole.

Increasing the pressure in the gas phase produces an increase in oxidizer concentration and, consequently, a decrease in the critical gap dimension: $L_{cr} \sim p^{-\alpha}$ ($\alpha > 1$; p is the pressure in the gas phase). Unlike the induction period, which is determined by the mass concentration [3], $L_{cr}$ is determined by the total surface of particles.

Indeed, the possibility of sustaining a steady-state mode depends on the ratio of incoming heat (determined by the surface of particles and the hot wall temperature $T_w$) to outgoing heat (determined by the thermal-conduction characteristics of the gas and the cold wall temperature $T_0$), while the induction period is related to the heat content.

The critical surface temperature (ensuring combustion) as a function of the particle radius, at a constant gap dimension, has a maximum and a minimum value. The maximum arises because the state of particles becomes unsteady earlier than the state of the system as a whole, while the minimum arises because the conditions become critical with respect to complete combustion [2, 3].

From the critical condition obtained for combustion during intensive homogenization of a mixture (small radii; particle and gas temperatures almost equal) there proceeds the result reported by Ya. B. Zel'dovich in [1].

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# A SUPERSONIC EJECTOR WITH A NOZZLE HAVING A VERY THICK DISCHARGE RIM*

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If the discharge rim of the central supersonic nozzle of an ejector has a considerable thickness, the active jet induces in the initial section of the mixing chamber a complex flow consisting of the ejecting jet and a circulating flow in the breakdown zone between the jets and the ground end. The sum of the ejector-current impulse in the initial section of the mixing chamber and the static ground impulse is taken as the value characterizing the inducing flow. It was established by experimental studies that the ratio of this sum to the impulse of the active flow (the relative inducing impulse) is almost independent of the ejection coefficient and the length of the mixing chamber in noncritical systems. The relative inducing impulse for the given injector proved to be a function only of the Mach number of the active nozzle and the ratio of the initial total pressures of the primary and secondary gases. An empirical expression is obtained for the relative inducing impulse in noncritical systems of ejector operation.

# INCIDENCE OF AN INDETERMINATE SUPERSONIC JET ON A PLANE†

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UDC 533.601.15

When a supersonic indeterminate jet is incident on a plane mounted perpendicular to the nozzle axis, there forms in front of it a receding shock wave, the form of development of which depends on the distance between the nozzle and the plane. The case of small distances was examined in [2]. If the distance between the nozzle and the plane is such that the receding shock wave enters the region of rarefaction of the free jet, where flow away from the source is realized [1, 3], then its curvature is reversed toward the plane and it interacts with the standing discontinuity of the jet. Having written discharge equations for the gas current up to the receding shock wave and beyond it, using a function for the shock wave and the assumption that the density and velocity component parallel to the plane are constant beyond the receding shock wave, it is possible to obtain a normal differential equation which allows the geometrical form of the development of the receding shock wave to be determined.

The derivative of the shock wave equation in a triple configuration of shock waves, which is determined from its calculation, serves as the boundary condition for this equation. In addition, the value of this derivative allows the coordinate of the triple shock wave configuration to be determined from an ordinary algebraic function.

All the gas dynamic parameters in the subsonic region of flow beyond the shock wave are also determined for a known form and position of the wave relative to the plane.

Calculations of the coordinates of the triple point according to the system presented, as well as the geometrical form of the receding shock wave, were compared with the results obtained after analysis of shadow photographs of the flow. Good agreement of the calculated and experimental data was established.

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† Leningrad Mechanical Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 21, No. 5, pp. 941-942, November, 1971. Original article submitted June 24, 1970; abstract submitted February 22, 1971.

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# BREAK-AWAY DIAMETER FREQUENCY OF A BUBBLE DURING BOILING UNDER CONDITIONS OF FREE CONVECTION

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An analytical solution for the break-away diameter of a vapor bubble, obtained in [1], taking into account the inertial force of the liquid mass, referred to the movement of the growing bubble, has the form

$$\frac{d_0}{l_*} = C_0 [8\varphi(\theta)]^{\frac{1}{2}}. \tag{1}$$

Here $C_0$ is a specific function of the dimensionless parameter

$$K = \sqrt{8}\,\frac{c_m\beta^2}{[\varphi(\theta)]^{\frac{3}{2}}} \cdot \frac{\rho' a'^2}{\sigma l_*}\,\mathrm{Ja}^2.$$

The growth rate of the bubble is taken from [2]:

$$\frac{dR}{d\tau} = \beta\,\frac{a'}{R}\,\mathrm{Ja}. \tag{2}$$

At a bubble break-away frequency $f = c_\tau/\tau_1$ a comparison of Eqs. (1) and (2) gives

$$f = \frac{\sqrt{K}}{C_0^2}\,\frac{c_\tau}{[8c_m^2\varphi(\theta)]^{\frac{1}{4}}}\left[\frac{\sigma}{\rho' l_*^3}\right]^{\frac{1}{2}}. \tag{3}$$

A comparison of the solutions (1) and (3) with experimental results showed that the solutions obtained correctly reflect the nature of the functions under consideration and give satisfactory quantitative correspondence with the experimental data at large values of the parameter K (for water Ja $\geq$ 30).

It was found that the experimental results corresponding to small values of the criterion Ja deviate from the solutions (1) and (3), and possible reasons for this deviation were indicated.

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# BUBBLING IN A HOMOGENOUS WATER PULSED REACTOR *

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In the article, a model of the formation of nuclei of the gaseous phase by means of fission fragments is studied in application to a homogeneous water reactor. A fragment produces $\delta$ electrons along its path. As a result of the local generation of heat caused by the braking of the $\delta$ electrons, a continuous vapor trail is produced along the track of the fragment and then breaks up into finer individual bubbles.

Moreover, in its flight the fragment produces radiolysis of the water into hydrogen and oxygen along the track. It is assumed that this "radiolytic" gas remains within the boundaries of the vapor trail and then within the bubbles. A bubble formed in the liquid by a fission fragment has a temperature higher than that of the surrounding liquid. Upon cooling, it contracts to the point where the pressure within it does not satisfy the Laplace relation for a stable radius assuming equality of temperature in the bubble and in the liquid. Here the pressure in the bubble is determined by the sum of the partial pressures of the radiolytic gas and the water vapor.

The pressure dynamics of the bubble (its growth or contraction) is determined by the diffusion of gas through its surface, which depends on the concentration of radiolytic gas in solution in the active zone at the time of formation of the bubble.

A system of equations is developed to describe the process of radiolytic boiling in the active zone of the reactor during a pulse. The concentration of radiolytic gas at the boundary of a bubble in the liquid is determined from Henry's law for the solubility of gases. It is assumed that the concentration gradient at the boundary of the bubble has the form

$$\operatorname{grad} C = \sqrt{\frac{3}{\pi}} \cdot \frac{C_\infty - C_0}{\sqrt{Dt}},$$

where $C_0$ and $C_\infty$ are the concentrations of radiolytic gas in the liquid at the bubble boundary and far from it, respectively; D is the diffusion coefficient; t is the time.

The results of calculations for several given pulses are presented. It is found that the nature of the boiling depends essentially on the initial pressure in the reactor and on the number of bubbles formed per fragment (i.e., on the size of these bubbles). It is shown that the start of boiling at the bottom of the reactor lags behind the start of boiling at the surface of the active zone.

# MEASUREMENT OF SMALL MECHANICAL OSCILLATIONS USING THE MÖSSBAUER EFFECT †

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UDC 53.082.79

A method of high energetic resolution, based on the Mössbauer effect, allows the detection of minute energy variations with a precision of up to $10^{-6}$ eV.

The possibility of applying the Mössbauer effect to measure mechanical oscillations of small amplitude on the order of $10^{-4}$–$10^{-2}$ $\mu$ in the ultrasonic frequency range of 20–100 kHz is examined in the article.

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It is shown that the transition can be made from low-frequency to high-frequency measurements of the amplitude and velocity of mechanical oscillations. It is also shown that averaging of the resonance absorption effect with respect to the velocity does not depend on the vibration frequency, but on the maximum velocity of the mechanical oscillations. For this reason, amplitude vibrometers using this effect are extremely simple in construction. In them $Sn^{119}O_2$ (a gamma source) and $SnO_2$ (an absorber at room temperature) can be used as the resonance couple. The determination of vibration velocities and amplitudes reduces to comparative measurements of the impulse counting rate behind the absorber (in a transmission geometry) and the counting rate without the absorber.

A calibration graph of the dependence of the resonance absorption effect on the maximum velocity was calculated theoretically on the basis of the Breit–Wigner equation. According to the graph the total measurement error averages ~10%.

The sensitivity (lower limit) and range of the velocity and amplitude measurements of vibrations using a Mössbauer amplitude vibrometer depend on the instrumental absorption line width. In accordance with this two subranges of the oscillation amplitude measurements are isolated: $10^{-4}$–$10^{-3}$ and $10^{-3}$–$10^{-2}$ $\mu$.

From the evaluation presented it is concluded that the sensitivity of a Mössbauer vibrometer is an order of magnitude greater than that obtained with the interference method.

# ON SATISFYING THE MOISTURE DISTRIBUTION FUNCTION IN AN INORGANIC MEDIUM, ARISING FROM AN ELEMENTARY PLANE SOURCE, BY THE NONLINEAR DIFFERENTIAL EQUATION OF MOISTURE CONDUCTION

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On the basis of a moisture distribution function in an inorganic medium from an instantaneous plane source, which is an exact solution of a linear differential equation, an impulse method for determining the coefficient of moisture diffusion is developed. It is shown in the article that the given function is an approximate solution of a nonlinear differential equation of moisture conduction under isothermal conditions expressing the dependence of the diffusion coefficient on the moisture content.

The deviation of the linear differential equation from the nonlinear equation is evaluated on the basis of experimental data obtained for 22 materials. The error with which the moisture diffusion coefficient is determined by the impulse method neglecting its dependence on the moisture content is presented: it has the average value 17%. Moreover, the given function permits the calculation of the exact coefficient of moisture diffusion, taking into account its variation due to the moisture content.

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# A REFINEMENT OF THE INTEGRAL HEAT-BALANCE EQUATION AND ITS APPLICATION TO PROBLEMS CONSIDERED IN NONLINEAR HEAT-CONDUCTION THEORY*

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We consider the one-dimensional nonlinear heat-conduction equation

$$C(\theta)\frac{\partial\theta}{\partial \mathrm{Fo}}=\frac{\partial}{\partial x}\left[\Lambda(\theta)\frac{\partial\theta}{\partial x}\right], \tag{1}$$

for an approximate solution of which we apply the integral relation

$$\frac{d}{d\,\mathrm{Fo}}\int_0^\delta C\theta dx-\int_0^\delta\frac{\partial C}{\partial\,\mathrm{Fo}}\theta dx-C\theta\Big|_{x=\delta}\frac{d\delta}{d\,\mathrm{Fo}}=\Lambda\frac{\partial\theta}{\partial x}\Big|_{x=\delta}-\Lambda\frac{\partial\theta}{\partial x}\Big|_{x=0}, \tag{2}$$

which is the thermal analogue of the dynamic Karman–Pohlhausen relation.

The main source of error in the given method is associated with the presence of the derivative of the unknown function $\theta(x, \mathrm{Fo})$ in the second term on the right side of the integral relation (2). As the result of two consecutive quadratures of the initial Eq. (1), this term may be written in terms of integrals of the unknown function

$$-\Lambda\frac{\partial\theta}{\partial\,\mathrm{Fo}}\Big|_{x=0}=\frac{1}{\delta}\left\{\int_0^\delta\left[\frac{\partial}{\partial\,\mathrm{Fo}}\int_0^x C\theta dx\right]dx-\int_0^\delta\left[\int_0^x\frac{dC}{\partial\,\mathrm{Fo}}\theta dx\right]dx+\int_{\theta|_{x=\delta}}^{\theta|_{x=0}}\Lambda d\theta\right\}. \tag{3}$$

Thanks to the integral form of representing the derivative of the unknown function $\theta(x, \mathrm{Fo})$, the method is less sensitive to the choice of the function approximating the temperature distribution in the thermal layer $\delta(\mathrm{Fo})$.

The more precise method may be illustrated by solving the problem of the heating of a body with variable thermophysical properties when it is subjected to boundary conditions of the first kind in the case of nonstationary and quasistationary modes.

# TEMPERATURE DISTRIBUTION IN A TWO-DIMENSIONAL CONDUCTING LAYER WITH A VARIABLE CURRENT †

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The distribution of the steady temperature t and the current in the plate $|z| \le 1$ which is heated by a monochromatic current $I\exp(i\omega\tau)$ may be determined from the equations and boundary conditions

$$t''=-\frac{a^2}{2\lambda\sigma}|j|^2,\quad t(1)=t'(0)=0;\quad j''=2in^2j,\quad j'(0)=0,\quad \int_0^1 j\,dz=\frac{I}{2a}, \tag{1}$$

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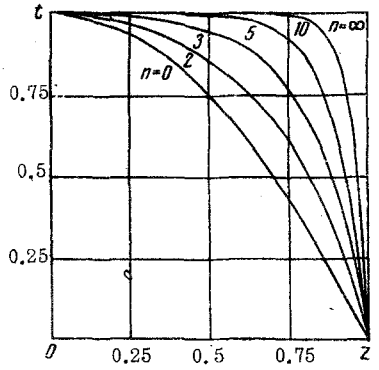


Fig. 1. Temperature distribution in a two-dimensional layer for various values of the number n (for various current frequencies).

where z is a coordinate referred to the plate semithickness $a$; $\lambda$, $\sigma$, and $\mu$ are, respectively, its thermal conductivity, electrical conductivity, and magnetic permeability; $n \equiv a\sqrt{\mu\sigma\omega/2}$; j is the complex amplitude of the current density. From this we have

$$j = \frac{I}{2a}(1+i)\,n\,\frac{\operatorname{ch} 2(1+i)nz}{\operatorname{sh} 2(1+i)n}, \tag{2}$$

or

$$|j| = \frac{I}{2a}n\sqrt{2\,\frac{c_1(2nz)}{c_2(2n)}},\quad \arg j = \frac{\pi}{4} - [\operatorname{arctg}(\operatorname{cth} n \operatorname{tg} n) + \operatorname{arctg}(\operatorname{th} nz \operatorname{tg} nz)]; \tag{3}$$

$$t = \frac{I^2}{16\lambda\sigma}\left[1 - \frac{c_2(2nz)}{c_2(2n)}\right];\quad \bar{t} = \frac{I^2}{16\lambda\sigma}\left[1 - \frac{s_2(2n)}{2nc_2(2n)}\right],\quad t(0) = \frac{I^2}{16\lambda\sigma} \tag{4}$$

(see Fig. 1), where $c_{1,2}(x) = \operatorname{cth} x \pm \cos x$, $s_{1,2}(x) = \sinh x \pm \sin x$. At small frequencies ($n \ll 1$) we obtain a small correction to the distribution $t_0 = (16\lambda\sigma)^{-1}I^2(1 - z^2)$ ($\bar{t}_0 = I^2/24\lambda\sigma$) for a constant current $I\sqrt{2}$; at high frequencies ($n \gg 1$), t is almost constant everywhere except in the vicinity of surface layers of thickness of order $a/2n$, where it falls sharply (in the limit as $n \to \infty$, $t = (I^2/16\lambda\sigma)\operatorname{sign}(1 - |z|)$):

$$t \simeq t_0\left[1 + \frac{2n^4}{45}z^2(1+z^2)\right],\quad \bar{t} \simeq \bar{t}_0\left(1 + \frac{4n^4}{315}\right)\ (n < 0{,}5);\quad t \simeq t(0)\{1 - \exp[-2n(1-z)]\},$$
$$\bar{t} \simeq t(0)\left(1 - \frac{1}{2n}\right)\ (n > 3). \tag{5}$$

We note that $t \geq t_0$, $\partial t/\partial n \geq 0$, $\partial t(0)/\partial n = 0$, $(t/t_0)' \geq 0$, $t/t_0(1) = ns_1(2n)/c_2(2n)$ (which is close to $1 + 4n^4/45$ and to n, respectively, for these same limiting cases).

Thus, along with an absolute electromagnetic skin effect, there is a relative temperature skin effect, i.e., the relative redistribution of the temperature in the use of the near-surface regions is at the expense of the depth. This is natural since, with an increase in frequency, the principal sources of Joule heat are all concentrated in a small neighborhood of the surfaces of the two-dimensional layer.

The redistribution of the temperature in the plate with an increase in n (in the frequency) is analogous to the evolution of the relative profile of the local mean velocity of the liquid in a two-dimensional channel with an increase in the Reynolds number (in the outflow rate). This analogy is universal for all forms of conductor cross sections and, correspondingly, of the channel cross sections for a liquid.

# NONSTATIONARY HEAT CONDUCTION IN BODIES WITH A UNIFORMLY MOVING HEAT-EMITTING LAYER

G. G. Golov

UDC 536.21

In this paper we consider an unbounded or a semibounded body with an initial temperature $T_0$. The thermal characteristics of the body material, i.e., the coefficients of thermal conductivity ($\lambda$) and thermal diffusivity ($a_1^2$), are assumed to be constant.

From the instant of time $\tau = 0$ a two-dimensional heat-emitting layer of thickness $a$ and with source density $(A + BT_2)\lambda/a_1^2$ moves along the body; A and B are constants of arbitrary sign, the first of which may be zero.

The rate of displacement u of the layer is constant with time.

For the semibounded body, conditions of thermal insulation or of constant temperature are imposed on the moving bounding surface, which is at the same time a boundary of the heat-emitting layer.

The heat-conduction equations for an unbounded body in a moving system of coordinates have the form

$$\frac{\partial T_1}{\partial \tau} = a_1^2 \frac{\partial^2 T_1}{\partial \xi^2} + u \frac{\partial T_1}{\partial \xi} \quad (\tau > 0,\ \xi > 0);$$

$$\frac{\partial T_2}{\partial \tau} = a_1^2 \frac{\partial^2 T_2}{\partial \xi^2} + u \frac{\partial T_2}{\partial \xi} + A + BT_2 \quad (\tau > 0,\ -a < \xi < 0);$$

$$\frac{\partial T_3}{\partial \tau} = a_1^2 \frac{\partial^2 T_3}{\partial \xi^2} + u \frac{\partial T_3}{\partial \xi} \quad (\tau > 0,\ \xi < -a);$$

$$T_1(\xi, \tau)|_{\tau=0} = 0,\ (\xi > 0);\ T_2(\xi, \tau)|_{\tau=0} = 0 \quad (-a < \xi < 0);\ T_3(\xi, \tau)|_{\tau=0} = 0 \quad (\xi < -a);$$

$$T_1(\xi, \tau)|_{\xi=\infty} = 0,\ T_3(\xi, \tau)|_{\xi=-\infty} = 0,\ T_1(\xi, \tau)|_{\xi=0} = T_2(\xi, \tau)|_{\xi=0},\ T_2(\xi, \tau)|_{\xi=-a} =$$

$$= T_3(\xi, \tau)|_{\xi=-a},\ \left.\frac{\partial T_1(\xi, \tau)}{\partial \xi}\right|_{\xi=0} = \left.\frac{\partial T_2(\xi, \tau)}{\partial \xi}\right|_{\xi=0},\ \left.\frac{\partial T_2(\xi, \tau)}{\partial \xi}\right|_{\xi=-a} = \left.\frac{\partial T_3(\xi, \tau)}{\partial \xi}\right|_{\xi=-a},\ (\tau > 0).$$

The temperature of the semibounded body may be described by the equations

$$\frac{\partial T_1}{\partial \tau} = a_1^2 \frac{\partial^2 T_1}{\partial \xi^2} + u \frac{\partial T_1}{\partial \xi} \quad (\tau > 0,\ \xi > 0);$$

$$\frac{\partial T_2}{\partial \tau} = a_1^2 \frac{\partial^2 T_2}{\partial \xi^2} + u \frac{\partial T_2}{\partial \xi} + A + BT_2 \quad (\tau > 0,\ -a < \xi < 0);$$

$$T_1(\xi, \tau)|_{\tau=0} = 0;\ (\xi > 0);\ T_2(\xi, \tau)|_{\tau=0} = 0 \quad (-a < \xi < 0);\ T_1(\xi, \tau)|_{\xi=\infty} = 0, \tag{1}$$

$$\left.\frac{\partial T_2(\xi, \tau)}{\partial \xi}\right|_{\xi=-a} = 0 \text{ or } T_2(\xi, \tau)|_{\xi=-a} = T_S - T_0,$$

$$T_1(\xi, \tau)|_{\xi=0} = T_2(\xi, \tau)|_{\xi=0},\ \left.\frac{\partial T_1(\xi, \tau)}{\partial \xi}\right|_{\xi=0} = \left.\frac{\partial T_2(\xi, \tau)}{\partial \xi}\right|_{\xi=0} (\tau > 0),$$

where $T_S$ is the temperature of the body on the bounding surface.

After making the substitutions

$$T_1(\xi, \tau) = T_1'(\xi, \tau) \exp[(B - u^2/4a_1^2)\tau],\quad T_2(\xi, \tau) = T_2'(\xi, \tau) \exp[(B - u^2/4a_1^2)\tau],$$

$$T_3(\xi, \tau) = T_3'(\xi, \tau) \exp[(B - u^2/4a_1^2)\tau]$$

we can solve the heat-conduction equations by an operational method.

The inverse Laplace transform may be reduced to a contour integral (see Fig. 1), which may be evaluated with the help of Jordan's lemma and Cauchy's residue theorem. The temperature transforms have simple poles at $S = u^2/4a_1^2 - B$. When $B > 0$ a finite number of poles of the first order also appears on the interval $(-B, 0)$ at the points $(-X_n)$. In the unbounded body problem these points may be determined from equation

$$\operatorname{ctg}(aa_1^{-1}\sqrt{X_n}) = -(B - 2X_n)/2\sqrt{X_n}\sqrt{B - X_n},$$

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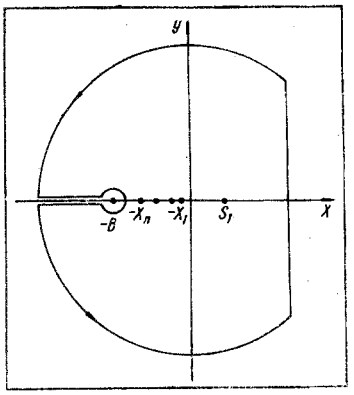


Fig. 1. Contour of integration in the plane of the complex variable S for $B > 0$: $S = -B$ is a branch point; $-X_n$ and $S_1 = u^2/4a_1^2$ are first order poles.

and, in problems for a body with the boundary conditions (1), from the equation

$$\operatorname{ctg}(aa_1^{-1}\sqrt{X_n}) = -\left(\sqrt{B-X_n}\frac{u}{2a_1}-X_n\right)\left[\sqrt{X_n}\left(\frac{u}{2a_1}+\sqrt{B-X_n}\right)\right]^{-1}$$

or

$$\operatorname{tg}(aa_1^{-1}\sqrt{X_n}) = -\frac{\sqrt{X_n}}{\sqrt{B-X_n}}.$$

If, for all $X_n$ (corresponding to the problem), the inequality $(B - X_n - (u^2/4a_1^2) < 0$ is satisfied, then the temperature of the body tends towards a steady distribution as $\tau$ increases. The values of the temperature $|T(\xi, \tau)|$ increase in time without bound when even for one $X_n$ this condition is not satisfied.

When $B \leq 0$ a steady distribution necessarily exists.

The solution of the heat conduction equations for the unbounded body simplify greatly if $B = 0$:

$$T_1(\xi, \tau) = \frac{A}{2}\int_0^\tau\left[\operatorname{erf}\left(\frac{a+\xi}{2a_1\sqrt{\theta}}+\frac{u\sqrt{\theta}}{2a_1}\right)-\operatorname{erf}\left(\frac{\xi}{2a_1\sqrt{\theta}}+\frac{u\sqrt{\theta}}{2a_1}\right)\right]d\theta,$$

$$T_2(\xi, \tau) = \frac{A}{2}\int_0^\tau\left[\operatorname{erf}\left(\frac{a+\xi}{2a_1\sqrt{\theta}}+\frac{u\sqrt{\theta}}{2a_1}\right)-\operatorname{erf}\left(\frac{\xi}{2a_1\sqrt{\theta}}+\frac{u\sqrt{\theta}}{2a_1}\right)\right]d\theta,$$

$$T_3(\xi, \tau) = \frac{A}{2}\int_0^\tau\left[\operatorname{erf}\left(\frac{a+\xi}{2a_1\sqrt{\theta}}+\frac{u\sqrt{\theta}}{2a_1}\right)-\operatorname{erf}\left(\frac{\xi}{2a_1\sqrt{\theta}}+\frac{u\sqrt{\theta}}{2a_1}\right)\right]d\theta.$$

(In fact, these equations can be used for the approximate calculation of the temperature of the body for small $\tau$ and $B \neq 0$).

In the paper the results of an illustrative calculation of the temperature for an unbounded body is presented.

# APPROXIMATE DETERMINATION OF THERMOELASTIC STRESSES IN A SPHERE DURING SYMMETRICAL HEATING

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UDC 539.377

An approximate method of determining the thermoelastic stresses in a hollow sphere ($R_1 \leq r \leq R_2$) heated symmetrically from the outside is examined. The process of nonsteady-state heat conduction is interpreted by an engineering model. During the inertial stage, while the preheating is taking place, the cross section of the sphere is divided into two zones: a preheated zone ($R_1 + b(t) \leq r \leq R_2$) with an excess temperature $T_1(r, t)$ and an unpreheated zone ($R_1 \leq r \leq R_1 + b(t)$) with zero excess temperature. Here b(t) is the thickness of the unpreheated zone. The general solution of the thermoelastic problem in the inertial stage has the following form:

$$\sigma_r^{(0)} = \frac{2\alpha E}{1-\mu} \cdot \frac{r^3 - R_1^3}{(R_2^3 - R_1^3) r^3} \int_{R_1+b(t)}^{R_2} T_1 r^2 dr;$$

$$\sigma_\theta^{(0)} = \frac{\alpha E}{1-\mu} \cdot \frac{2r^3 + R_1^3}{(R_2^3 - R_1^3) r^3} \int_{R_1+b(t)}^{R_2} T_1 r^2 dr;$$

$$\sigma_r^{(1)} = \frac{2\alpha E}{1-\mu} \cdot \frac{1}{r^3} \left[ \frac{r^3 - R_1^3}{R_2^3 - R_1^3} \int_{R_1+b(t)}^{R_2} T_1 r^2 dr - \int_{R_1+b(t)}^{r} T_1 r^2 dr \right];$$

$$\sigma_\theta^{(1)} = \frac{\alpha E}{1-\mu} \cdot \frac{1}{r^3} \left[ \frac{2r^3 + R_1^3}{R_2^3 - R_1^3} \int_{R_1+b(t)}^{R_2} T_1 r^2 dr + \int_{R_1+b(t)}^{r} T_1 r^2 dr - T_1 r^3 \right],$$

where $\sigma^{(0)}$ and $\sigma^{(1)}$ are the stresses in the unpreheated and preheated zones, respectively; $\alpha$ is the linear expansion coefficient; E is the elastic modulus; $\mu$ is the Poisson coefficient.

During the regular heating stage, when the temperature varies through the entire cross section of the body, the thermoelastic stresses are determined by well-known expressions which can be obtained from the equations for $\sigma^{(1)}$, setting b(t) = 0 and substituting the temperature function corresponding to this stage. As an example a calculation is presented of the stresses in a solid sphere with a uniform initial temperature, heated at a constant surface temperature. Approximate temperature functions are obtained by the method of averaging functional corrections. It is shown that in the inertial stage, in the unpreheated zone, the radial and circumferential stresses are positive, equal in value, and constant in coordinate. The maximum tension stresses arise in the center of the sphere at the moment the inertial stage ends, corresponding to the greatest temperature drop across the cross section for the given boundary conditions of heating. In the regular stage, when temperature equalization takes place, the stresses decrease. Graphs are presented illustrating the solution obtained. The results of the calculation are compared with the exact solution.

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# HEAT CONDUCTION IN SOLID OR HOLLOW CYLINDERS WITH HEAT-EXCHANGE COEFFICIENTS DEPENDING ON THE ANGLE

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In [1, 2] the heat-conduction problem for massive solid cylinders,

$$\frac{\partial t(\rho, \varphi, \mathrm{Fo})}{\partial \mathrm{Fo}} = \nabla^2 t(\rho, \varphi, \mathrm{Fo}) + R_2^2 L(\mathrm{Fo})\, t(\rho, \varphi, \mathrm{Fo}) + \frac{R_2^2}{\lambda} Q(\rho, \varphi, \mathrm{Fo}), \tag{1}$$

$$\frac{\partial t(\omega, \varphi, \mathrm{Fo})}{\partial \rho} - [\mathrm{Bi}_1(\varphi) + \overline{\mathrm{Bi}}]\, t(\omega, \varphi, \mathrm{Fo}) = -\mathrm{Bi}_1(\varphi)\, t_{c1}(\varphi, \mathrm{Fo}) - \frac{\overline{\mathrm{Bi}}}{4} \int_{\varphi}^{2\pi+\varphi} t(\omega, \theta, \mathrm{Fo}) \sin\frac{\theta-\varphi}{2}\, d\theta, \tag{2}$$

$$-\frac{\partial t(1, \varphi, \mathrm{Fo})}{\partial \rho} + \mathrm{Bi}_2(\varphi)\,[t_{c2}(\varphi, \mathrm{Fo}) - t(1, \varphi, \mathrm{Fo})] = 0 \tag{3}$$

and for thin-wall hollow cylinders,

$$\frac{\partial t(\varphi, \mathrm{Fo})}{\partial \mathrm{Fo}} = \frac{\partial^2 t(\varphi, \mathrm{Fo})}{\partial \varphi^2} - M(\varphi, \mathrm{Fo})\, t(\varphi, \mathrm{Fo}) + Q(\varphi, \mathrm{Fo}) - \frac{\omega\, \overline{\mathrm{Bi}}}{1-\omega}\left[ t(\varphi, \mathrm{Fo}) - \frac{1}{4} \int_{\varphi}^{2\pi+\varphi} t(\theta, \mathrm{Fo}) \sin\frac{\theta-\varphi}{2}\, d\theta \right], \tag{4}$$

supplemented by the initial conditions, the conditions of temperature periodicity and of continuity of the heat flow with respect to the angle condition, were solved under the assumptions $\mathrm{Bi}_k = \mathrm{const}$ ($k = 1, 2$) and $M(\varphi, \mathrm{Fo}) = M(\mathrm{Fo})$. The analogous problem for a solid cylinder is formulated in the form (1), (3) replacing (2) by the condition of the finiteness of $t(0, \varphi, \mathrm{Fo})$ [3]. In Eqs. (1)-(4) $\omega = R_1, R_2$ are the inner and outer cylinder radii. The other notation is conventional.

If the heat-exchange coefficients are dependent on the angle (then $\mathrm{Bi}_k$ and M also depend on $\varphi$) it is very difficult to find the kernels of the integral transformations. The solution of the problem (1)-(3) and (4) was therefore obtained by means of perturbation theory. In the case of constant $\mathrm{Bi}_k$ and $M(\varphi, \mathrm{Fo}) = M(\mathrm{Fo})$ the problem remains unperturbed. If

$$\mathrm{Bi}_k(\varphi) = \mathrm{Bi}_{k0}\,[C_k + \beta\, \mathrm{Bi}_{k1}(\varphi)], \tag{5}$$

$$M(\varphi, \mathrm{Fo}) = M_0(\mathrm{Fo})\,[C + \beta M_1(\varphi, \mathrm{Fo})], \tag{6}$$

$$\max|\mathrm{Bi}_{k1}(\varphi)| \leqslant C_k,\ \max|M_1(\varphi, \mathrm{Fo})| \leqslant C,\ \beta < 1, \tag{7}$$

the solution is sought in the form of a power series

$$t(\rho, \varphi, \mathrm{Fo}) = \sum_{\nu=0}^{\infty} \beta^{\nu}\ t_{\nu}(\rho, \varphi, \mathrm{Fo}). \tag{8}$$

It has been shown that the first two conditions of (7) can always be satisfied. The substitution of (8) into the original equations results in recurrence relations for $t_\nu(\rho, \varphi, \mathrm{Fo})$ (for thin-walled pipes the sought function is independent of $\rho$), each being similar to the unperturbed problem. With (7) satisfied the the convergence of (8) is proved in the case of solid or hollow (massive or thin-walled) cylinders.

A numerical example has shown that for $\beta = 0.2$ it suffices that the first three terms of the series (8) be retained. The solutions thus obtained enable one to show the principal differences between two heating modes, in each of which either the temperature of the medium or the heat-exchange coefficient are angle-dependent.

In the paper the roots of the equation

$$\frac{I_m(\mu_{m,n})}{I_{m+1}(\mu_{m,n})} = \frac{\mu_{m,n}}{m + \mathrm{Bi}}$$

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were m, n = 1, 2, ..., 6 and $0 \leq Bi \leq 100$; furthermore, proposals were made for a new index characterizing the lack of symmetry of the heat flow and the functions $M(\varphi, Fo)$ and $Q(\varphi, Fo)$ were obtained.

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# TWO REPRESENTATIONS OF THE TEMPERATURE FIELD FOR GIVEN POWER FUNCTIONS OF TIME


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Cases are considered in which the boundary conditions and strength of heat sources are specified as power functions of time with exponents equal to 0, 0.5, 1, and 1.5. Special solutions are obtained for an infinite plate and a spherical shell using the Laplace transform of the fundamental solution consisting of thermal potentials. These solutions are then expressed in terms of integrals of the probability integral. It is shown that, by integration operations applied to the general representation given in a form which is more convenient for long intervals of time, special solutions can be obtained. These special solutions are expressed in terms of exponential trigonometric series and Bernoulli or Euler polynomials. The remainder of the series is estimated.



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